Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Thursday 18 May 2023

Afternoon (Time: 2 hours)

Paper reference **8MA0/01**

Mathematics

Advanced Subsidiary

PAPER 1: Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 17 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. A curve has equation

$$y = \frac{2}{3}x^3 - \frac{7}{2}x^2 - 4x + 5$$

(a) Find $\frac{dy}{dx}$ writing your answer in simplest form.

(2)

(b) Hence find the range of values of x for which y is decreasing.

(4)

a) To find dy/dx, differentiate y with respect to x.

remember $\boxed{\frac{d}{dx}\, ax^b = bax^{b-1}}$

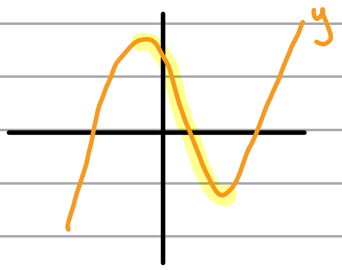
$y = \frac{2}{3}x^3 - \frac{7}{2}x^2 - 4x + 5$

$\frac{dy}{dx} = 3 \times \frac{2}{3}x^2 - 2 \times \frac{7}{2}x - 1 \times 4x^0$

$= \boxed{2x^2 - 7x - 4}$

$x^0 = 1$ so the derivative of $-4x$ is just -4

b) When y is decreasing, $\frac{dy}{dx}$ is negative.



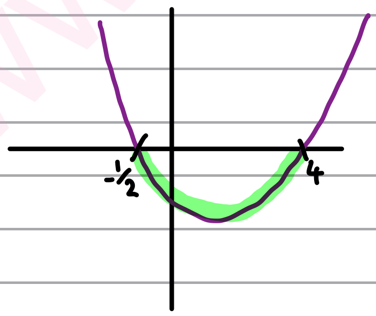
sketch the initial graph, and highlight where the gradient is negative. These are the x values we are trying to find.

Then solve $2x^2 - 7x - 4 < 0$ to find the values of x when the function is decreasing.

$2x^2 - 7x - 4 < 0$

factorise

$(2x+1)(x-4) < 0$



sketch the graph by finding the roots

Because we are finding $(2x+1)(x-4) < 0$, we need to find the range of values which are below the x axis.

the critical values are $x = -\frac{1}{2}$ and $x = 4$ (found by finding the roots)

Question 1 continued

so the range of values between these are

$$\boxed{-\frac{1}{2} < x < 4}$$

(Total for Question 1 is 6 marks)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Using the substitution $u = \sqrt{x}$ or otherwise, solve

$$6x + 7\sqrt{x} - 20 = 0$$

(4)

2. Solving $6x + 7\sqrt{x} - 20 = 0$ would be difficult without a substitution as it is not in a known form to solve. By using $u = \sqrt{x}$, we can convert the equation into a quadratic equation of u, because $(\sqrt{x})^2 = x$

$$6(\sqrt{x})^2 + 7(\sqrt{x}) - 20 = 0$$

Sub in $u = \sqrt{x}$ $\quad 6u^2 + 7u - 20 = 0$

Factorise to Solve for u $\quad (3u - 4)(2u + 5) = 0$

$u = 4/3 \quad u = -\frac{5}{2}$

$u = \sqrt{x}$

$\sqrt{x} = 4/3 \quad \sqrt{x} = -\frac{5}{2}$

we can't find the square root of a negative number, so $\sqrt{x} = -\frac{5}{2}$ cannot be a solution

$$\boxed{x = 16/9}$$

P 7 2 8 3 9 A 0 4 4 4

Question 2 continued

(Total for Question 2 is 4 marks)

3.

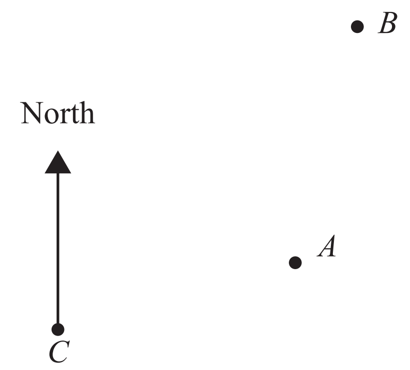


Figure 1

Figure 1 is a sketch showing the position of three phone masts, A, B and C.

The masts are identical and their bases are assumed to lie in the same horizontal plane.

From mast C

- mast A is 8.2 km away on a bearing of 072°
- mast B is 15.6 km away on a bearing of 039°

(a) Find the distance between masts A and B, giving your answer in km to one decimal place.

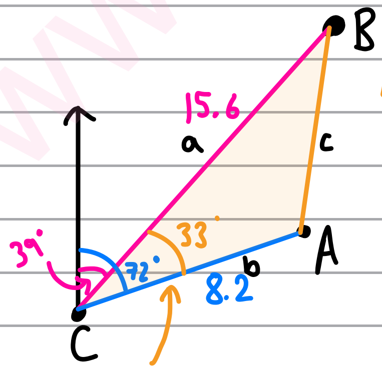
(3)

An engineer needs to travel from mast A to mast B.

(b) Give a reason why the answer to part (a) is unlikely to be an accurate value for the distance the engineer travels.

(1)

a) Draw a diagram to represent the information given in the question. Remember, a bearing is the angle in degrees measured clockwise from north.



The orange line AB is what we are trying to find.

As we have now formed a triangle with two sides known and one angle known, we can use the cosine rule to find the length AB.

angle $B\hat{C}A$ is $72° - 39°$

$= 33°$

Question 3 continued

where A is the angle opposite the missing side

The cosine rule is $a^2 = b^2 + c^2 - 2bc\cos A$

but our missing side is c, so swap the as and cs around (because the cos angle must be known)

$c^2 = b^2 + a^2 - 2ba\cos C$

sub in a, b and C to find c

$c^2 = 8.2^2 + 15.6^2 - (2 \times 8.2 \times 15.6 \times \cos 33)$

$= 67.24 + 243.36 - 214.565\ldots$

$c^2 = 96.0345\ldots$

$c = 9.7997\ldots$

$\boxed{c = 9.8\text{ km}}$

b) The road between A and B is unlikely to be straight, so the distance would be greater

(Total for Question 3 is 4 marks)

4. (a) Sketch the curve with equation

$$y = \frac{k}{x} \qquad x \neq 0$$

where k is a positive constant.

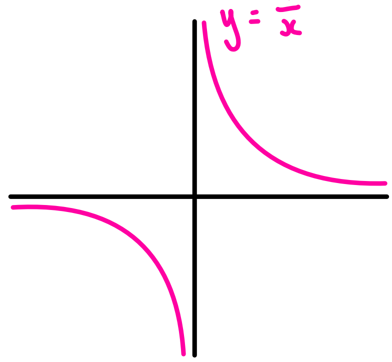
(2)

(b) Hence or otherwise, solve

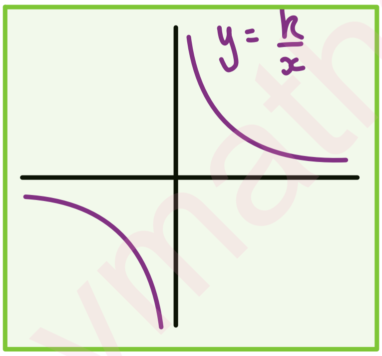
$$\frac{16}{x} \leqslant 2$$

(3)

a) $y = \frac{1}{x}$



so $y = \frac{k}{x}$ has the same shape

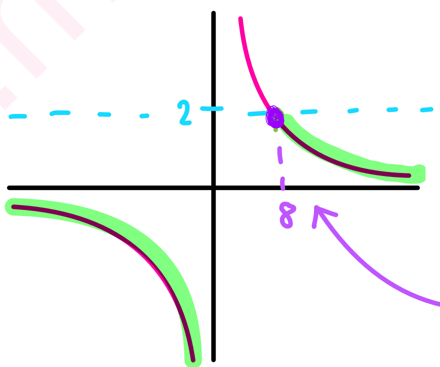


since k just multiplies each y value, hence you can't see a difference unless both graphs were drawn together

$k = 16$

b) $\frac{16}{x} \leqslant 2$

we want values less than or equal to 2



to find •

$\frac{16}{x} = 2$

$16 = 2x$

$x = 8$

We are looking for values equal or less to 2. We can see these by looking at the graph.

$\boxed{x < 0, \ x \geqslant 8}$

Question 4 continued

(Total for Question 4 is 5 marks)

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

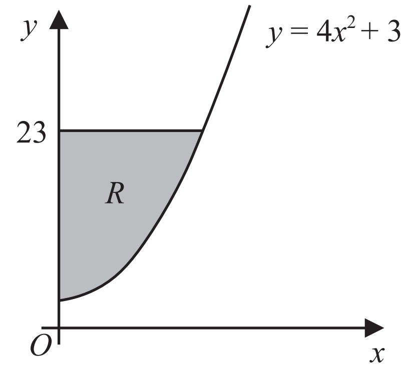


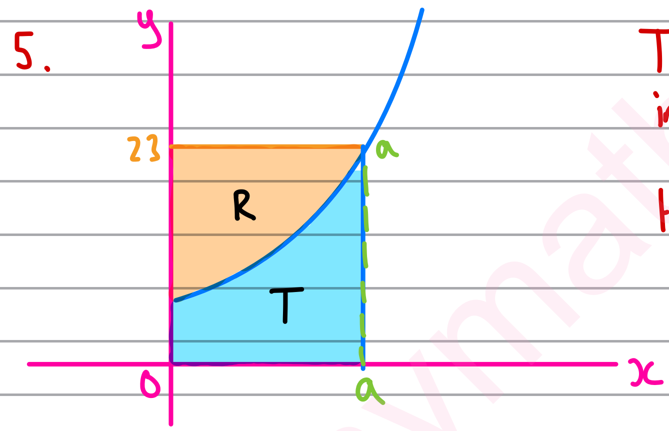
Figure 2

The finite region R, shown shaded in Figure 2, is bounded by the curve with equation $y = 4x^2 + 3$, the y-axis and the line with equation $y = 23$

Show that the exact area of R is $k\sqrt{5}$ where k is a rational constant to be found.

(5)

5.



To find the area of R, you must integrate the equation $y = 4x^2 + 3$

However you can't find the area of R directly: you have to find the area below the curve first (T), then subtract it from the area of the rectangle that contains it.

To find area of T: $\int_0^a 4x^2 + 3 \, dx$

the limit is unknown so we have to find a, which is the point on the line $y = 4x^2 + 3$ where $y = 23$, (see diagram)

$y = 23$
$y = 4x^2 + 3$

$23 = 4x^2 + 3$
$20 = 4x^2$
$5 = x^2 \qquad x = \pm\sqrt{5}$
$x = \sqrt{5}$

x must be $\sqrt{5}$ because we can see that it is positive from the diagram

so $a = \sqrt{5}$

Question 5 continued

$$T = \int_0^{\sqrt{5}} 4x^2 + 3 \; dx$$

integrate by adding one to the power, then dividing by the new power

$$\left[\tfrac{4}{3}x^3 + 3x \right]_0^{\sqrt{5}}$$

← it is not necessary to +c because we are integrating with limits

$$= \left(\tfrac{4}{3}\sqrt{5}^3 + 3\sqrt{5}\right) - \left(\tfrac{4}{3}\times 0 + 3\times 0\right)$$ ← sub in the limits $\sqrt{5}$ and 0

$$= \frac{20}{3}\sqrt{5} + 3\sqrt{5}$$

$$T = \frac{29}{3}\sqrt{5}$$

Now find the rectangle which contains R and T

$R + T =$ area of rectangle

$$R + \frac{29}{3}\sqrt{5} = 23\sqrt{5}$$

$$R = 23\sqrt{5} - \frac{29}{3}\sqrt{5}$$

$$R = \frac{69}{3}\sqrt{3} - \frac{29}{3}\sqrt{5}$$

$$\boxed{R = \frac{40}{3}\sqrt{5}}$$

(Total for Question 5 is 5 marks)

6. The circle C has equation

$$x^2 + y^2 - 6x + 10y + k = 0$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

Given that C does not cut or touch the x-axis,

(b) find the range of possible values for k.

(3)

a) To find the co-ordinates of the centre, the equation must be in the form: $(x+a)^2 + (y+b)^2 = r^2$

To do this, you have to complete the square of both x and y.

$$x^2 + y^2 - 6x + 10y + k = 0$$

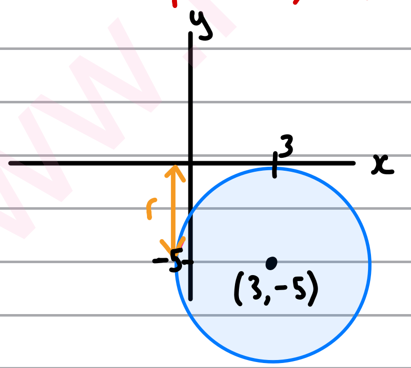
$$(x-3)^2 - 9 + (y+5)^2 - 25 + k = 0$$

$$(x-3)^2 + (y+5)^2 = 34 - k$$

the co-ordinates of the centre are $(-a, -b)$

$$\boxed{\text{centre} = (3, -5)}$$

b) To solve part b, draw a diagram of the circle



To not touch the x axis, the maximum radius of the circle is just under 5 . $r < 5$

the radius must be greater than 0 always $r > 0$

$$0 < r < 5$$

When finding the equation of the circle the radius was $\sqrt{34-k}$

$r^2 = 34 - k \qquad r = \sqrt{34 - k}$

P 7 2 8 3 9 A 0 1 2 4 4

Question 6 continued

so, $\sqrt{34-k} < 5$ and $\sqrt{34-k} > 0$

$34-k < 25$ $\qquad$ $34-k > 0$

$9 < k$ $\qquad$ $34 > k$

$\boxed{9 < k < 34}$

(Total for Question 6 is 5 marks)

7. The distance a particular car can travel in a journey starting with a full tank of fuel was investigated.

- From a full tank of fuel, 40 litres remained in the car's fuel tank after the car had travelled 80 km
- From a full tank of fuel, 25 litres remained in the car's fuel tank after the car had travelled 200 km

Using a **linear model**, with V litres being the volume of fuel remaining in the car's fuel tank and d km being the distance the car had travelled,

(a) find an equation linking V with d.

(4)

Given that, on a particular journey

- the fuel tank of the car was initially full
- the car continued until it ran out of fuel

find, according to the model,

(b) (i) the initial volume of fuel that was in the fuel tank of the car,

(ii) the distance that the car travelled on this journey.

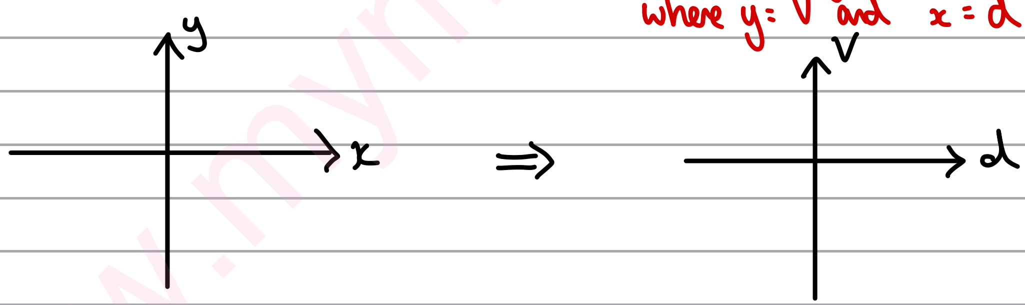
(3)

In fact the car travelled 320 km on this journey.

(c) Evaluate the model in light of this information.

(1)

a) As it is a linear model, use the format $y = mx + c$
where $y = V$ and $x = d$



$V = md + c$

Then sub in the two examples given in the question

1) $V = 40$ $d = 80$
2) $V = 25$ $d = 200$

① $40 = 80m + c$
② $25 = 200m + c$

P 7 2 8 3 9 A 0 1 4 4 4

Question 7 continued

Then to find the complete model, solve the two simultaneous equations to find c and m

$2.5 \times ①: \quad 100 = 200m + 2.5c$

$②: \quad 25 = 200m + c$

$2.5 \times ① - ② \quad 75 = 1.5c$

$c = 50$

$② \quad 25 = 200m + 50$

$-25 = 200m$

$-25/200 = m$

$m = -1/8 \qquad m = -1/8 \quad c = 50$

Sub in $m = -1/8$ and $c = 50$ into the original equation

$\boxed{V = -\tfrac{1}{8}d + 50 \quad \text{or} \quad 8V = -d + 400}$

b) i) $V = -\tfrac{1}{8}d + 50$

when the car has travelled 0km $d = 0$

$V = 0 + 50 \quad \boxed{V = 50 \text{ Litres}}$

ii) when the car has run out of fuel, $V = 0$

$0 = -\tfrac{1}{8}d + 50$

$50 = \tfrac{1}{8}d \quad \boxed{d = 400 \text{ km}}$

c) bad model since 320 km is quite a bit less than 400 km

(Total for Question 7 is 8 marks)

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8.

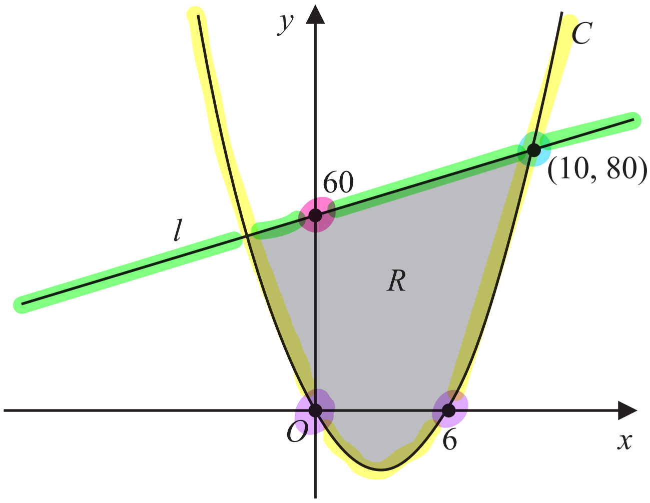

Figure 3

Figure 3 shows a sketch of a curve C and a straight line l.

Given that

- C has equation $y = f(x)$ where $f(x)$ is a quadratic expression in x
- C cuts the x-axis at 0 and 6
- l cuts the y-axis at 60 and intersects C at the point $(10, 80)$

use inequalities to define the region R shown shaded in Figure 3.

(5)

As curve C is a quadratic equation, it will be in the form

$y = ax^2 + bx + c$

The line l is a straight line so its equation is in the form:

$y = mx + c$

Now we need to sub in the points given in the question to find complete equations for C and l.

Start with l: We have been given two points on the graph, so we can find the equation of line l.

co-ordinates: $(10, 80)$, $(0, 60)$

To find gradient: $\frac{y_1 - y_2}{x_1 - x_2} = \frac{80-60}{10-0} = \frac{20}{2} = 2$ $m = 2$

P 7 2 8 3 9 A 0 1 6 4 4

Question 8 continued

to find L

$y = 2x + c$

Sub in (10, 80) : $80 = 2 \times 10 + c$

$80 = 20 + c$

$c = 60$

$y = 2x + 60$

Now find C: $y = ax^2 + bx + c$

sub in (0,0) $c = 0$ as the y intercept $= 0$

so $y = ax^2 + bx$

Find C in factorised form:

look at diagram to see x-intercepts

The roots are 0 and 6, so the equation can be written as

$y = a(x-0)(x-6)$ $y = ax(x-6)$

we still need to find a:

when (10, 80)

$80 = 10a(x-6)$

$8 = 4a$

$a = 2$

$C = 2x(x-6)$

By looking at the graph, the shaded area R is greater than curve C, and less than line L, so:

$$\boxed{2x(x-6) \leq y \leq 2x + 60}$$

$\leq$ is necessary due to the solid lines surrounding R.

(Total for Question 8 is 5 marks)

9. Using the laws of logarithms, solve the equation

$$2\log_5(3x-2) - \log_5 x = 2$$

(5)

To solve this equation, all parts of the equation should be in the same form, log base 5. ($\log_5$)

$$2\log_5(3x-2) - \log_5 x = 2$$

$$2\log_5(3x-2) - \log_5 x = \log_5 25$$

$2 = \log_5 25$ because $5^2 = 25$

1

$\boxed{a\log_b x = \log_b x^a}$

Formula to learn

2

$\boxed{\log_a x - \log_a y = \log_a \frac{x}{y}}$

$$\downarrow ①$$

$$\log_5(3x-2)^2 - \log_5 x = \log_5 25$$

$$\downarrow ②$$

$$\log_5 \frac{(3x-2)^2}{x} = \log_5 25$$

Now that both sides are in the form $\log_5$, we can remove the $\log_5$ from each and solve for x

$$\frac{(3x-2)^2}{x} = 25$$

$$(3x-2)^2 = 25x$$

$$9x^2 - 12x + 4 = 25x$$

$$9x^2 - 37x + 4 = 0$$

$$(9x-1)(x-4) = 0$$

$$x = 4 \quad x = 1/9$$

$x = 1/9$ cannot be a solution, because logs cannot be taken from a negative number. If $x = 1/9$ was subbed into the equation, the first term would be: $2\log_5(-5/3)$

$$\boxed{\therefore x = 4 \text{ only}}$$

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Question 9 continued

(Total for Question 9 is 5 marks)

10.

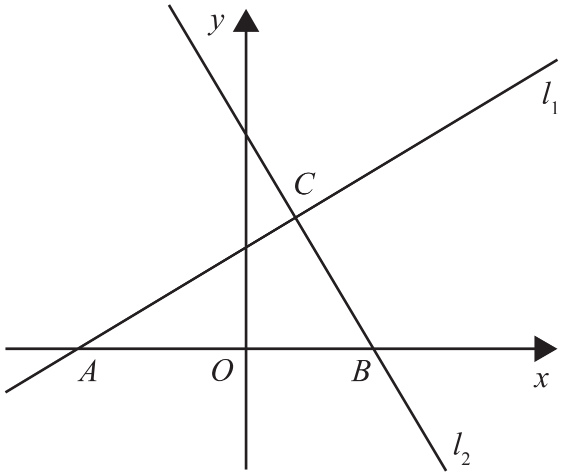

Figure 4

The line l_1 has equation $y = \frac{3}{5}x + 6$

The line l_2 is perpendicular to l_1 and passes through the point $B(8,0)$, as shown in the sketch in Figure 4.

(a) Show that an equation for line l_2 is

$$5x + 3y = 40$$

(3)

Given that

- lines l_1 and l_2 intersect at the point C
- line l_1 crosses the x-axis at the point A

(b) find the exact area of triangle ABC, giving your answer as a fully simplified fraction in the form $\frac{p}{q}$

(5)

a) When two lines are perpendicular, the product of their gradients is -1.

gradient of $l_2 = -\frac{5}{3}$, because $\frac{3}{5} \times -\frac{5}{3} = -1$

Now sub in (8,0) and $m = -\frac{5}{3}$ into $y = mx + c$ to find an equation of l_2.

gradient of l_2

$y = mx + c \rightarrow y = -\frac{5}{3}x + c$

$0 = -\frac{5}{3} \times 8 + c$

sub in (8,0)

Question 10 continued

$0 = -\frac{40}{3} + c \quad \therefore c = \frac{40}{3}$

$y = -\frac{5}{3}x + \frac{40}{3}$ Then rearrange to get the required form.

$3y = -5x + 40$

$\boxed{3y + 5x = 40}$

b) To find the area of a triangle, you must find the base and height:

$\boxed{\text{area} = \frac{1}{2} \times \text{base} \times \text{height}}$

1. Find the base by finding A.

y co-ordinate of A = 0.

sub $y=0$ into $y = \frac{3}{5}x + 6$ to find x co-ordinate:

$0 = \frac{3}{5}x + 6$

$0 = 3x + 30$

$3x = -30$

$x = -10$ so $A: (-10, 0)$

$B = (8, 0)$ so the length of the base is 18.

2. Find the height by finding the y co-ordinate of C, the intersection of l_1 and l_2. Solve l_1 and l_2 simultaneously.

Rearrange l_1 to make the subject x:

$\frac{3}{5}x = y - 6$

$x = \frac{5}{3}y - 10$

Then sub into l_2 to eliminate x:

$5\left(\frac{5}{3}y - 10\right) + 3y = 40$

$\frac{25}{3}y - 50 + 3y = 40$

$\frac{34}{3}y = 90$

Question 10 continued

$$34y = 270$$

$$y = \frac{270}{34} = \frac{135}{17}$$

$$\boxed{\text{area} = \tfrac{1}{2} \times \text{base} \times \text{height}}$$

$$\text{area} = \tfrac{1}{2} \times 18 \times \frac{135}{17}$$

$$\boxed{= \frac{1215}{17}}$$

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DO NOT WRITE IN THIS AREA

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Question 10 continued

(Total for Question 10 is 8 marks)

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Turn over

11. The height, h metres, of a plant, t years after it was first measured, is modelled by the equation

$$h = 2.3 - 1.7e^{-0.2t} \qquad t \in \mathbb{R} \quad t \geqslant 0$$

Using the model,

(a) find the height of the plant when it was first measured,

(2)

(b) show that, exactly 4 years after it was first measured, the plant was growing at approximately 15.3 cm per year.

(3)

According to the model, there is a limit to the height to which this plant can grow.

(c) Deduce the value of this limit.

(1)

a) When it was first measured, $t=0$

$h = 2.3 - 1.7e^{0}$ ← $e^{0} = 1$

$= 2.3 - 1.7$

$\boxed{= 0.6\text{m}}$

b) To find the rate the plant grows at, differentiate the height with respect to time (rate is synonymous with differentiation)

$\frac{dh}{dt} = -0.2 \times -1.7e^{-0.2t}$ ← $\boxed{\frac{dy}{dx}\, e^{ax} = ae^{ax}}$

$= 0.34e^{-0.2t}$

sub in $t = 4$

$\frac{dh}{dt} = 0.34e^{(-0.2\times 4)}$

$= 0.34e^{-0.8}$

$= 0.153\text{m}$

$\boxed{= 15.3\text{cm per year}}$

c) To find the limit, sub in a large value of t into $h = 2.3 - 1.7e^{0.2t}$ to find the maximum value of h

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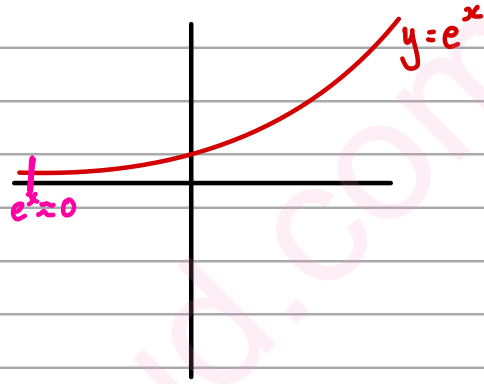
Question 11 continued

$h = 2.3 - 1.7e^{-0.2 \times 999}$

$\boxed{= 2.3\text{m}}$

graph of e^x. As x gets very negative, e^x converges to 0



(Total for Question 11 is 6 marks)

12. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$4\tan x = 5\cos x$$

can be written as

$$5\sin^2 x + 4\sin x - 5 = 0$$

(3)

(b) Hence solve, for $0 < x \leqslant 360°$

$$4\tan x = 5\cos x$$

giving your answers to one decimal place.

(4)

(c) Hence find the **number of solutions** of the equation

$$4\tan 3x = 5\cos 3x$$

in the interval $0 < x \leqslant 1800°$, explaining briefly the reason for your answer.

(2)

a) Use identities

1 $\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$

2 $\boxed{\sin^2\theta + \cos^2\theta = 1}$

1. remove tan by using $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$4\tan x = 5\cos x$

1 $\frac{4\sin x}{\cos x} = 5\cos x$

2. use identity $\sin^2 x + \cos^2 x = 1$ → $\cos^2 x = 1 - \sin^2 x$ to get rid of $\cos^2 x$

$4\sin x = 5\cos^2 x$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

2

$4\sin x = 5(1 - \sin^2 x)$

$4\sin x = 5 - 5\sin^2 x$

3. rearrange to get required form.

$\boxed{5\sin^2 x + 4\sin x - 5 = 0}$

P 7 2 8 3 9 A 0 2 6 4 4

Question 12 continued

b) To solve $4\tan x = 5\cos x$, solve $5\sin^2 x + 4\sin x - 5 = 0$ because we have just shown they are the same.

Replace $\sin x$ with y and solve the quadratic

$$5y^2 + 4y - 5 = 0$$

Use quadratic formula to solve

$$\boxed{y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$y = \frac{-4 \pm \sqrt{16 + 100}}{10}$$

$$= \frac{-4 \pm \sqrt{116}}{10} = \frac{-4 \pm 2\sqrt{29}}{10}$$

$$= \frac{-2 \pm \sqrt{29}}{5}$$

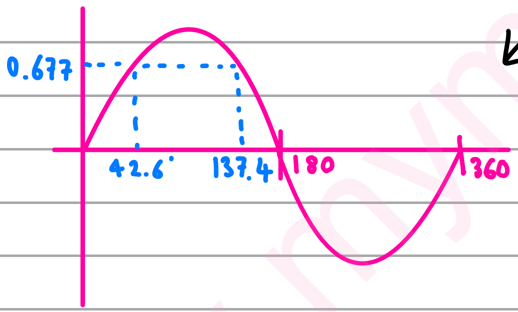
$$\sin x = \frac{-2 \pm \sqrt{29}}{5}$$

$\sin x = 0.677$ $\quad \cancel{= -1.477}$

$\sin x$ must be between -1 and 1

Find x by taking $\sin^{-1}$

$x = 42.6°$



$x = 180 - 42.6$
$= 137.4°$

There are two solutions between 0 and 360.

$$\boxed{x = 42.6° \text{ and } 137.4°}$$

c) There are two solutions in 360° (in the first equation), so in 1800 there will be 10. Changing the x to $3x$ will increase the number of solutions in that range by a factor of 3, as the curve is stretched ('squished') parallel to the x axis.

Question 12 continued

P 7 2 8 3 9 A 0 2 8 4 4

Question 12 continued

(Total for Question 12 is 9 marks)

13. Relative to a fixed origin O

- point A has position vector $10\mathbf{i} - 3\mathbf{j}$
- point B has position vector $-8\mathbf{i} + 9\mathbf{j}$
- point C has position vector $-2\mathbf{i} + p\mathbf{j}$ where p is a constant

(a) Find $\overrightarrow{AB}$

(2)

(b) Find $\left|\overrightarrow{AB}\right|$ giving your answer as a fully simplified surd.

(2)

Given that points A, B and C lie on a straight line,

(c) (i) find the value of p,

(ii) state the ratio of the area of triangle AOC to the area of triangle AOB.

(3)

a) To find $\overrightarrow{AB}$, subtract vector A from vector B, because we always do 2nd letter minus 1st.

$(-8i + 9j) - (10i - 3j)$

$= -8i + 9j - 10i + 3j$

$\overrightarrow{AB} = -18i + 12j$

b) $|\overrightarrow{AB}|$ is the magnitude of the vector $\overrightarrow{AB}$. Use pythagoras to find the magnitude.

$|\overrightarrow{AB}| = \sqrt{(-18)^2 + (12^2)}$

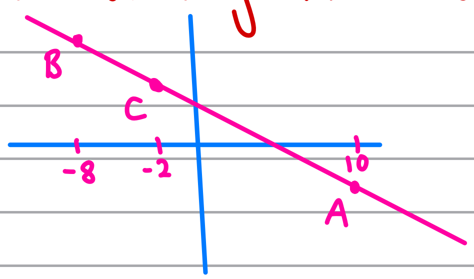
$= \sqrt{324 + 144}$

$= \sqrt{468}$

$= \sqrt{36}\sqrt{13}$

$\boxed{= 6\sqrt{13}}$

c) i) Draw a diagram to visualise the problem:



← we can picture where C is on the line by using the i co-ordinate given

Question 13 continued

where λ is some multiple of x

$\overrightarrow{AB} = \lambda \overrightarrow{AC}$

$\overrightarrow{AC} = C - A$

$-18i + 12j = \lambda(-2i + pj - (10i - 3j))$

$-18i + 12j = \lambda(-2i + pj - 10i + 3j)$

$-18i + 12j = -2\lambda i + p\lambda j - 10\lambda i + 3\lambda j$

because i and j are perpendicular they can be solved separately.

$-18 = -2\lambda - 10\lambda$

$-18 = -12\lambda$

$\boxed{\lambda = 3/2}$

$12 = p\lambda + 3\lambda$

$12 = 3/2\, p + 9/2$

$15/2 = 3/2\, p$

$15 = 3p$

$\boxed{p = 5}$

then solve them to find λ and then p

ii)

$\overrightarrow{AB} = 3/2\, \overrightarrow{AC}$

from earlier in the question $\overrightarrow{AB} = \lambda \overrightarrow{AC}$

$3\overrightarrow{AC} = 2\overrightarrow{AB}$

so ratio of side lengths AC : AB is 2:3

so ratio of triangle AOC to triangle AOB is $\boxed{2:3}$

Question 13 continued

Question 13 continued

(Total for Question 13 is 7 marks)

14. Find, in simplest form, the coefficient of x^5 in the expansion of

need x^3 term from 2nd bracket

$$\left(5+8x^2\right)\left(3-\frac{1}{2}x\right)^6$$

need x^5 term from 2nd bracket

(5)

14. $\boxed{x^a \times x^b = x^{a+b}}$

so we need to find: coefficient of x^3 in expansion of $(3-\frac{1}{2}x)^6$ to multiply by $8x^2$ $\rightarrow$ gets x^5

coefficient of x^5 in expansion of $(3-\frac{1}{2}x)^6$ to multiply by 5 $\rightarrow$ gets x^5

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Use the binomial expansion to expand $(3-\frac{1}{2}x)^6$

$1\times 3^6 \times(-\frac{1}{2}x)^0 + 6\times 3^5\times(-\frac{1}{2}x)^1 + 15\times 3^4\times(-\frac{1}{2}x)^2 + \boxed{20\times 3^3\times(-\frac{1}{2}x)^3} + 15\times 3^2\times(-\frac{1}{2}x)^4$
$+ \boxed{6\times 3\times(-\frac{1}{2}x)^5} + 1\times 3^0\times(-\frac{1}{2}x)^6$

the two parts we need

$= \ldots + -\frac{135}{2}x^3 + \ldots + -\frac{9}{16}x^5 + \ldots$

Then multiply by $(5+8x^2)$ to find the required powers of 5.

	$-\frac{135}{2}x^3$	$-\frac{9}{16}x^5$
5	$-\frac{675}{2}x^3$	$-\frac{45}{16}x^5$
$8x^2$	$-540x^5$	$-\frac{9}{2}x^7$

these are the two we need as the power of x is 5.

Add them together to get the final coefficient of x^5

$$-540-\frac{45}{16} = \boxed{-\frac{8685}{16}}$$

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Question 14 continued

(Total for Question 14 is 5 marks)

15. **In this question you must show detailed reasoning.**

Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation $y = 8 - 10x + 6x^2 - x^3$

The curve C_2 has equation $y = x^2 - 12x + 14$

(a) Verify that when $x = 1$ the curves C_1 and C_2 intersect.

(2)

The curves also intersect when $x = k$.

Given that $k < 0$

(b) use algebra to find the exact value of k.

(5)

a) When the question says verify, you can just sub in the values given in the question

when $x=1$ $\quad y = 8 - 10(1) + 6(1^2) - 1^3$
$= 8 - 10 + 6 - 1$
$= 3$

$y = 1^2 - 12(1) + 14$
$= 1 - 12 + 14$
$= 3$

when $x=1$, $y=3$ for both curves.
(1,3) lies on both curves so they intersect at $x=1$

b) If the curves intersect again, the y values are the same

$8 - 10x + 6x^2 - x^3 = x^2 - 12x + 14$
(we will replace x with k later)

$x^3 - 5x^2 - 2x + 6 = 0$

this is a cubic so we cannot solve by factorising or the quadratic equation. However we already know $(x-1)$ is a factor because we have already shown $x=1$ is a solution.

dividing by $x-1$ to find a quadratic we can solve

	x^2	$-4x$	-6
x	x^3	$-4x^2$	$-6x$
-1	$-x^2$	$4x$	6

$(x-1)(x^2 - 4x - 6) = 0$

now we have a quadratic we can solve to find a second value of x. Solve using the quadratic equation

P 7 2 8 3 9 A 0 3 6 4 4

Question 15 continued

$$x = \frac{4 \pm \sqrt{16 - 4\times1\times{-6}}}{2}$$

← use quadratic equation to find x

$$= \frac{4 \pm \sqrt{40}}{2}$$

$$= \frac{4 \pm 2\sqrt{10}}{2}$$

$$x = 2 \pm \sqrt{10}$$

x must be less than 0 (given in question)

We also know $x = k$

so $\boxed{k = 2 - \sqrt{10} \text{ only}}$

Question 15 continued

Question 15 continued

(Total for Question 15 is 7 marks)

16. A curve has equation $y = f(x),\ x \geqslant 0$

Given that

- $f'(x) = 4x + a\sqrt{x} + b$, where a and b are constants
- the curve has a stationary point at $(4,3)$
- the curve meets the y-axis at -5 $\longrightarrow$ $(0,-5)$

find $f(x)$, giving your answer in simplest form.

(6)

16. To find a and be, find two equations containing a and b to solve simultaneously

Because $(4,3)$ is a stationary point, $f'(x) = 0$

when $x = 4$

$0 = 4 \times 4 + a\sqrt{4} + b$

$0 = 16 + 2a + b$ ⟵ first equation

Then integrate $f'(x)$, to find $f(x)$ in terms of a and b

$f(x) = \int 4x + ax^{\frac{1}{2}} + b$

$= 2x^2 + \frac{2}{3}ax^{3/2} + bx + c$

$= 2x^2 + \frac{2}{3}ax^{3/2} + bx - 5$ ⟵ $c = -5$ because we are given that the y intercept is -5 in the question

sub in $(4,3)$ to find a 2nd equation

$3 = 2\times 4^2 + \frac{2}{3}a \times 4^{3/2} + 4b - 5$

$3 = 32 + \frac{16}{3}a + 4b - 5$

$0 = 24 + \frac{16}{3}a + 4b$ ⟵ 2nd equation

Solve simultaneously to find a and b:

① $0 = 16 + 2a + b$ ⟶ $4\times$① $0 = 64 + 8a + 4b$

② $0 = 24 + \frac{16}{3}a + 4b$ $\quad 4b = -64 - 8a$

sub into equation 2 to find a

$0 = 24 + \frac{16}{3}a - 64 - 8a$

$-40 = \frac{8}{3}a$ $\quad \underline{a = -15}$

$\underline{b = 14}$

Then sub in to find $f(x)$

$$\boxed{f(x) = 2x^2 - 10x^{\frac{3}{2}} + 14x - 5}$$

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Question 16 continued

(Total for Question 16 is 6 marks)

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17. In this question p and q are positive integers with $q > p$

Statement 1: $q^3 - p^3$ is never a multiple of 5

(a) Show, by means of a counter example, that Statement 1 is **not** true.

(1)

Statement 2: When p and q are consecutive **even** integers $q^3 - p^3$ is a multiple of 8

(b) Prove, using algebra, that Statement 2 is true.

(4)

a) Find an example (using trial and error) where $q^3 - p^3$ is a multiple of 5

when $q = 6$, $p = 1$

$6^3 - 1^3 = 216 - 1 = 215$

215 is a multiple of 5

b) when p and q are consecutive integers $p = 2n$, $q = 2n + 2$

Then show $q^3 - p^3$ is a multiple of 8

$(2n+2)^3 - (2n)^3 = 8n^3 + 24n^2 + 24n + 8 - 8n^3$

$= 24n^2 + 24n + 8$

expand using binomial expansion

$= 8(3n^2 + 3n + 1)$

So $q^3 - p^3$ is a multiple of 8

$1 \times (2n)^3 \times 2^0 + 3 \times (2n)^2 \times 2^1 + 3 \times (2n)^1 \times 2^2 + 1 \times (2n)^0 \times 2^3$

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Question 17 continued

Question 17 continued

(Total for Question 17 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS

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